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Angular Momentum Operator and Fermion-Pair Creation for Non-Abelian Fields

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We study the extended structure of non-Abelian dyons, the generalized electromagnetic field and the resulting residual angular momentum in the interior as well as exterior regions of the dyon, and it has been demonstrated that at the dyonic centre there exists no well-defined $U(1)$ charge symmetry and the density of residual angular momentum becomes infinity. The mechanism of creation of a fermionic pair at the dyonic core involving the extremely high density of residual angular momentum has been developed, which leads to baryon-number nonconservation in the presence of non-Abelian dyons. The fermion-number–breaking amplitudes in the presence of a non-Abelian dyon have been analyzed and are not suppressed by $\exp(-\text{const}/e^2)$. Further, the relevant properties of left-handed fermions in a non-Abelian field has been summarized and the zeroth-order approximation is described. Within this approximation the density of the fermion-number–breaking condensate is found to be $O(1)$, i.e. to be independent of the coupling constant and of the vacuum expectation value of the Higgs field.

KEY WORDS: dyon; residual angular momentum; dyonic core; fermion-pair creation; zeroth-order approximation.

1. INTRODUCTION

Though physicists were fascinated by the magnetic monopole since the ingenious idea was put forward by Dirac (1931, 1948) and Price *et al.* (1975, 1978) about its possible experimental verification, renewed interest in this subject was enhanced by the very strong argument of ’t Hooft (1974) and Polyakov (1974) that the spontaneously broken gauge theories with compact $U(1)$ gauge group guarantee the existence of smooth, topologically stable finite-energy solutions with quantized magnetic charge. Extending this, Julia and Zee (1975) constructed classical solutions having electric and magnetic charges on the same particle, called a dyon (Schwinger, 1966a,b).

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As such, the monopoles and dyons become an intrinsic part of all current grand-unified theories (Dokos and Tomaras, 1980; Preskill, 1984), with enormous potential importance in connection with their role in catalyzing baryon-number–nonconserving processes (Callen, 1982a,b; Rubakov, 1981a), the quark confinement problem of QCD (Mandelstam, 1976, 1979; 't Hooft, 1978) and RCD (Rajput *et al.*, 1989), the unification of gravitation with generalized electromagnetic field (Rajput, 1982, 1984; Rajput and Gunwant, 1988) and CP violation in terms of nonzero vacuum angle of world (Witten, 1979). Keeping in view the result of Witten that there are necessarily dyons, Rajput *et al.* (1982, 1986a,b) have constructed a self-consistent and covariant quantum field theory of dyons and demonstrated (Rajput *et al.*, 1989) the validity of two simultaneous recent experimental results about the existence of free fractionally charged quarks (Fairbank *et al.*, 1981) and monopoles (Cabrera, 1982) with magnetic charge of one Dirac unit. The dyon–fermion dynamics has been worked out by various authors (Mandelstam, 1976, 1979; Rajput *et al.*, 1989; Rubakov, 1981a), and it has been demonstrated that the helicity-conserving and charge-mixing boundary conditions to be imposed on fermionic fields at monopole core violate the charge superselection rule. A solution to this problem may lie in establishing the formation of chiral condensate, allowing the flipping of helicity and proving an understanding of the role of dyon as catalyst in baryon-number–nonconserving processes. This problem of chiral condensation at the dyonic core requires the explicit field solutions in the interior region of the dyon. Investigating the extended structure of non-Abelian dyons in the present paper, we have constructed suitable Lagrangian density and energy–momentum tensor in non-Abelian gauge theory of dyons, derived the expression of generalized electromagnetic field in the external and internal regions of extended dyon, and showed that there is no well-defined $U(1)$ charge symmetry at the dyonic centre. The contribution of these fields to the angular-momentum operator of non-Abelian dyon has been derived and the density of the resulting residual angular momentum has shown to be infinity at dyonic centre.

Not long since, the existence of 't Hooft–Polyakov magnetic monopole (Polyakov, 1974; 't Hooft, 1974) has been one of the most interesting features of spontaneous broken gauge theories. Most of the known characteristics of the 't Hooft–Polyakov magnetic monopole (mass, magnetic charge, etc.) manifest themselves already at the classical level, the quantum effects giving rise to $O(e^2)$ corrections (Jackiw, 1977). The only known exception is the deep relationship (Christ and Jackiw, 1980; Pak, 1980; Witten, 1979) between the magnetic charge and the winding number (Belavin *et al.*, 1975) of the gauge field. In theories without massless fermions this results in the Witten value of the charge of the quantum dyon (Witten, 1979). $Q_D = -e\theta/2\pi$, where θ is the CP nonconservation angle. In the present study we consider theories with massless left-handed fermions ($(V\text{-}A)$ theories). Our main purpose is to show that in these theories the above relationship leads to strong fermion-number–nonconservation in dyon–fermion interaction.

We also develop a suitable approximation for calculating some fermion-number–breaking matrix elements in the presence of a dyon.

It is well known that in $(V\text{-}A)$ theories the divergence of the (euclidean) fermionic current is anomalous (Adler *et al.*, 1969; Bell and Jackiw, 1969),

$$\partial_\mu J_\mu^F = \text{const} S_P F_{\mu\nu} \tilde{F}_{\mu\nu} = \text{const} E^a H^a,$$

so that the fermion number

$$N_{\mathrm{F}} = \int J_0^{\mathrm{F}}\, d^3x$$

is not conserved in the external field with a nonzero winding number q, where

$$q = -\frac{1}{32\pi^2}\varepsilon_{\mu\nu\lambda\rho} \int S_P F_{\mu\nu} F_{\lambda\rho}\, d^4x. \tag{1}$$

In the vacuum sector this effect is associated with instantons (Belavin *et al.*, 1975; Karasnikov *et al.*, 1978; Peccei and Quinn, 1977; 't Hooft, 1976a,b) and the fermion-number–breaking amplitudes are supressed by the factor $\exp(-\text{const}/e^2)$ as well as by negative powers of the vacuum expectation value of the Higgs field ('t Hooft, 1976a,b). The first supression results from the large values of action for the configurations with $q \neq 0$, while the second one results from the small value of the instanton size, which is cut off at the Compton length of the massive vector boson.

Since in the presence of non-Abelian dyons there exist a nonzero classical, electromagnetic field, $H^{\mathrm{cl}} \neq 0$ and $E^{\mathrm{cl}} \neq 0$ could give rise to the strong chirality breaking in quantum chromodynamics (Blaer *et al.*, 1991). The emission of light charged fermions from a Julia–Zee dyon was analyzed by Blaer *et al.* (1991); even for the case of massless fermions, this emission process violated chirality and provides a simple illustration of the axial anomaly. In the fermion-number–breaking amplitude there is no supression factor expected. One expects the anomalous fermion-number–breaking in the presence of dyons to be strong, presumably $O(1)$. This effect can have far-reaching consequences, the most interesting one being the strong baryon-number–nonconservation in fermion–dyon interactions in grand unified theories (Rubakov, 1981b,c).

From the above arguments it is clear that the actual calculation of Green functions with fermion-number breaking in the presence of a dyon will be rather nonstandard. The effect is neither perturbative (the $\bar{\psi}\psi A$ vertex conserves the fermion number) nor quasiclassical (since the factor $\exp(\text{const}/e^2)$ does not appear). This difficulty is inherent in the Schwinger model (Schwinger, 1962), where an exact solution (either operator (Krasnikov *et al.*, 1980a,b; Velo, 1967) or functional (Schwinger, 1962)) is needed to investigate the chirality and fermion-number breaking (Krasnikov *et al.*, 1980a,b; Lowenstein and Swieca, 1971; Nielsen and Schroer, 1977a,b; Rothe and Swieca, 1977, 1979; Schroer, 1978). Since we are unable to obtain an exact solution of the spontaneous, broken four-dimensional gauge theory, we are faced with the problem of developing a suitable zeroth-order approximation. However, the natural approximation does exist if we restrict ourselves to the dynamics of spherically symmetric fermions. Within this approximation one assumes the relevent gauge field configurations to be spherically symmetric and neglects the contribution of fermions with nonzero angular momentum to the fermionic determinant. Under these assumptions the problem becomes effectively two-dimensional and one can find an exact solution that is quite similar to the solution of the Schwinger model. The main part of the present paper is devoted to the description and solution of this approximation and to estimate corrections. Within this approximation it becomes possible to confirm the heuristic argument of Section. 3 and find the e^2 dependence of fermion-number–breaking matrix elements in the presence of a dyon.

2. GENERALIZED FIELD AND ANGULAR MOMENTUM ASSOCIATED WITH EXTENDED DYONS

Generalized fields associated with dyons of nonzero mass can be described only by a non-Abelian gauge theory consisting of usual four-space (external) and the n-dimensional internal group space. In such a theory, the field has n-fold internal multiplicity and the multiplets of gauge field transform as a basis of adjoint representation of n-dimensional non-Abelian gauge symmetry group. In order to preserve the invariance of generalized field equations of motion for dyons under the local non-Abelian gauge transformations

$$\psi \rightarrow \psi' = S^{-1}\psi, \tag{2.1}$$

with S a local group element of $SU(2)$, a generalized potential $\vec{V}_\mu$ is introduced, and derivatives of ψ are identified as covariant derivatives

$$\vec{\nabla}_\mu = (\vec{\partial}_\mu - iq^* \vec{V}_\mu \times), \tag{2.2}$$

where the vector sign $\rightarrow$ and cross product $\times$ are denoted in internal group space, $\mu = 0, 1, 2, 3,$ and are indices representing external degrees of freedom and the component V_μ^a of $\vec{V}_\mu$ in the internal space is a generalized four-potential given by

$$V_\mu^a = A_\mu^a - iB_\mu^a, \tag{2.3}$$

with A_μ^a and B_μ^a as electric and magnetic four-potentials associated with the dyons carrying the generalized charge

$$q = e - ig \tag{2.4}$$

in terms of electric and magnetic charges e and g respectively. It has already been shown (Rajput *et al.*, 1983) that for the fields associated with a massless dyon, the additional potential (B_μ) does not lead to an increase in the number of independent variables describing the fields, and the introduction of this potential is actually compensated by an enlargement of the group of gauge transformations. In non-Abelian gauge theory of massive dyons, $\vec{V}_\mu$ may be treated as a 2×2 Hermitian matrix and the generalized field tensor may be written as

$$\vec{G}_{\mu\nu} = \vec{V}_{\mu,\nu} - \vec{V}_{\mu,\nu} + iq^*[\vec{V}_\mu, \vec{V}_\nu]. \tag{2.5}$$

For extended dyons, it is possible (Wu and Yang, 1975) to divide the space–time manifold into two overlapping regions R_1 and R_2 such and B_μ vanishes in R_1 and A_μ vanishes in R_2.

In the internal two-dimensional complex space introduced at each point of Minkowski space–time, the charged field described by $SU(2)$ is replaced by $\exp\lfloor i\Lambda^O(x)\rfloor\psi$ in $SU(2) \times U(1)$, where $\Lambda^O(x)$ is a phase factor. Then the basic spinors of this internal space are acted upon by the following element S of $SU(3)$:

$$\vec{S}(x) = S(x)\,\exp\lfloor -i\Lambda^O(x)\rfloor \tag{2.6}$$

where $S(x)$ is a local group element of $SU(2)$ defined by Eq. (2.1). Under this gauge transformation the 2×2 matrix potential $\vec{V}_\mu$ and the matrix field tensor $\vec{G}_{\mu\nu}$ transform in the following manner:

$$\vec{V}_\mu = \vec{S}^{-1}\vec{V}_\mu\vec{S} - \vec{S}^{-1}\vec{\partial}_\mu\vec{S},$$

and

$$\vec{G}_{\mu\nu} = \vec{S}^{-1}\vec{G}_{\mu\nu}\vec{S}, \tag{2.7}$$

respectively. Instead of matrices $\vec{V}_\mu$ and $\vec{G}_{\mu\nu}$, we may define the gauge potential V_μ^a and the gauge field strength $G_{\mu\nu}^a$ as follows:

$$\vec{V}_\mu - V_\mu^a\vec{T}_a,$$

and

$$\vec{G}_{\mu\nu} = G_{\mu\nu}^a\vec{T}_a, \tag{2.8}$$

where the repeated indices for $SU(2)$ group are summed over 1, 2, and 3 (internal degrees of freedom) and the matrices $\vec{T}_a$, describing the infinitesimal generators of the group $SU(2)$, satisfy the commutation relation

$$(T_a, T_b) = i\varepsilon^{abc}T^c, \tag{2.9}$$

with ε^{abc} as the structure constants of the internal group.

Then Eq. (2.5) may be written as

$$G_{\mu\nu}^a \equiv \partial_\nu V_\mu^a + \partial_\mu V_\nu^a + q^*\varepsilon^{abc}V_{\mu b}V_{\nu c}, \tag{2.10}$$

which gives the relationship between the gauge potential V_μ^a and the non-Abelian gauge field tensor $G_{\mu\nu}^a$. The covariant derivative of this field tensor may then be written as

$$\nabla^\nu G^a_{\mu\nu} = \partial^\nu G^a_{\mu\nu} + iq^* \varepsilon^{abc} V^\nu_b G_{\mu\nu c} = J^a_\mu, \tag{2.11}$$

where

$$J^a_\mu - J^a_\mu + iq^* \varepsilon^{abc} V^\nu_b G_{\mu\nu c}, \tag{2.11a}$$

with

$$J^a_\mu - \partial^\nu G^a_{\mu\nu} - j^a_\mu - ik^a_\mu, \tag{2.12}$$

representing the generalized linear (Abelian) electric and magnetic four-currents j^a_μ and k^a_μ associated with the dyon. The non-Abelian field in Eq. (2.11) is Lorentz-covariant and reduces to the field equations derived earlier (Bhakuni and Rajput, 1982) in the Abelian limit when the structure constant ε^{abc} vanishes. The Notherian current j^a_μ given by Eq. (2.12) is manifestly conserved while the generalized non-Abelian current given by Eq. (2.11a) is not so but it satisfies the following generalized conservation law (i.e. gauge covariance):

$$\vec{\nabla}^\mu \cdot \vec{J}_\mu = 0, \tag{2.13}$$

where the ordinary derivative is replaced by covariant derivative.

In non-Abelian gauge theory the local gauge transformations of generalized potential may be generalized into the following form:

$$\vec{V}'_\mu \rightarrow \vec{V}_\mu + \nabla_\mu \vec{\Lambda}_q(x), \tag{2.14}$$

i.e.

$$\vec{V}'_\mu \rightarrow \vec{V}_\mu + \partial_\mu \vec{\Lambda}_q(x) + iq^*[\vec{V}_\mu \times \vec{\Lambda}_q(x)]$$

where the gauge term $\vec{\Lambda} q(x)$ is given by the following equations in terms of electric and magnetic gauge constituents:

$$\vec{\Lambda}_q(x) = \vec{\Lambda}_e(x) - i\vec{\Lambda}_g(x). \tag{2.15}$$

These results led to the following gauge changes in the generalized fields and currents for the invariance of field equations in non-Abelian gauge theory

$$\vec{G}_{\mu\nu} = iq^*[\vec{G}_{\mu\nu} \times \vec{\Lambda}_q(x)],$$

and

$$\vec{J}_\mu = iq^*[\vec{J}_\mu \times \vec{\Lambda}_q(x)], \tag{2.16}$$

which also shows that the generalized particle current, though not conserved in non-Abelian theory, is gauge-covariant and subsequently the field equations (2.11) are invariant to a nontrivial local gauge group. Substituting these gauge changes into Eq. (2.11a), one may readily realize that the Notherian current j_μ receives a contribution from gauge fields and hence it is not gauge-covariant.

Suitable Lagrangian density yielding field Eq. (2.11) can be written in the following form:

$$L = -\frac{1}{4}g^{\mu\rho}g^{\nu\rho}\vec{G}^{*}_{\mu\nu}\cdot\vec{G}_{\rho\sigma} + \frac{1}{2}g^{\mu\nu}\vec{V}^{*}_{\mu}\cdot\vec{J}_{\nu} - \frac{1}{2}g^{\mu\nu}\vec{\nabla}_{\mu}(\phi)\cdot\vec{\nabla}_{\nu}(\phi) - \eta V(\phi), \tag{2.17}$$

$$\nabla_{\mu}\phi_{a} = \partial_{\mu}\phi_{a} + q^{*}\varepsilon^{abc}V^{b}_{\mu}\phi^{c}, \tag{2.17a}$$

and

$$V(\phi) = \frac{1}{4}(\phi^{2}\phi_{a})^{2} - \frac{1}{2}\nu^{2}(\phi^{a}\phi_{a})$$
$$\nu^{2} = \mu^{2}/\eta, \tag{2.17b}$$

where $\nu^2 - \langle\phi\rangle^2$ determines the vacuum expectation value of the triplet Higgs field ϕ^a. This Lagrangian density leads to the following form of energy–momentum tensor for generalized system of dyons in non-Abelian theory

$$\begin{aligned} T^{\mu}_{\nu} = &-\left\{g^{\mu\alpha}g^{\rho\beta}\vec{G}^{*}_{\alpha\beta}\cdot\vec{G}_{\nu\rho} - g^{\mu\rho}\vec{V}^{*}_{\nu}\cdot\vec{J}_{\rho}\right. \\ &\left. - \frac{1}{4}\delta^{\mu}_{\nu}g^{\rho\alpha}g^{\alpha\beta}\vec{G}^{*}_{\alpha\beta}\cdot\vec{G}_{\rho\sigma} + \frac{1}{2}\delta^{\mu}_{\nu}g^{\rho\sigma}\vec{V}^{*}_{\rho}\cdot\vec{J}_{\sigma}\right\} \\ &-\left\{G^{\mu\rho}\vec{\nabla}^{*}_{\rho}(\phi)\cdot(\phi) - \frac{1}{2}\delta^{\mu}_{\nu}G^{\mu\sigma}\nabla^{*}_{\rho}(\phi)\nabla_{\sigma}(\phi) - \delta^{\mu}_{\nu}\eta V(\phi)\right\} \end{aligned} \tag{2.18}$$

from which one may define the expressions for the Hamiltonian and momentum of the systems. It may readily be shown that this Lagrangian density is invariant under the nontrivial local gauge transformation (2.14) of the generalized potential, provided that the local gauge changes in the generalized field and current are given by Eq. (2.16). Lagrangian density given by Eq. (2.17) leads to the field equation (2.11) only in the region where the current contribution due to Higgs fields is negligible (i.e. Yang–Mills field obeys the free field equation). Let us consider the region in which Higgs fields contribute nonnegligible source current (i.e. the dyonic core) and assume the absence of Noetherian current defined by Eq. (2.12) in this region such that the second term of the Lagrangian density (2.17) (i.e. the interaction of generalized potential with generalized current) vanishes. Then the Lagrangian density (2.17) leads to the following field equations:

$$\nabla^{\nu}G^{a}_{\mu\nu} = iq^{*}\varepsilon^{abc}\phi_{b}\nabla_{\mu}\phi_{c}, \tag{2.19}$$

with

$$J^{a}_{\mu} = iq^{*}\varepsilon^{abc}V^{\nu}_{b}G_{\mu\nu c},$$

and

$$\nabla^{\mu}\nabla_{\mu}\phi^{a} = -\mu^{2}\phi^{a} + \lambda(\phi^{a}\phi_{\beta})\phi^{a}. \tag{2.20}$$

Equations (2.19) and (2.20) lead to the following relationship between the generalized field and potential of a dyon and the Higgs field

$$(\vec{\nabla}_{\mu}\vec{\phi}) \times \vec{\phi} - V^{\nu} \times \vec{G}_{\mu\nu}. \tag{2.21}$$

Under these conditions in the dyonic core we find that Eq. (2.18) leads to

$$\partial_{\mu}T^{\mu}_{\nu} = 0. \tag{2.22}$$

This gives the conservation of stress energy tensor inside the dyonic core. Using this conservation law one can compute (Joshuna *et al.*, 1978) the time rate of change of four-momentum, i.e. the force exerted on the core at any instant of time. In the static limit, when all time derivatives are zero, the field equations (2.19) and (2.20) take the following form:

$$\begin{cases} \partial^{j}G^{a}_{\mu j} + iq^{*}\varepsilon^{abc}V^{j}_{b}G_{\mu jc} = iq^{*}\varepsilon^{abc}\nabla_{\mu}\phi_{b}\phi_{c} \\ \partial^{j}\nabla_{j}\phi^{a}iq^{*}\varepsilon^{abc}V^{\mu b}\nabla_{\mu}\phi_{c} = \mu^{2}\phi^{a} + \lambda(\phi^{b}\phi_{b})\phi^{a}. \end{cases} \tag{2.23}$$

Considering the values of fields satisfying these equations as an initial data set, the time evolution of the fields is determined by the field equations (2.19) and (2.20) at the dyonic core such that the conservation equation (2.22) is valid.

It has been demonstrated in our earlier paper (Rajput *et al.*, 1983) that an Abelian dyon moving in the generalized field of another dyon carries a residual angular momentum (field contribution) besides its orbital and spin angular momenta. Consequently there exists a chirality-dependent multiplicity in the eigenvalues of this angular momentum operator. Non-Abelian gauge field is self-interacting one and carries the charge even in the absence of external current sources, as shown by Eq. (2.19). Because of the interaction with this charged field, the non-Abelian dyon carries the residual angular momenta J_{res} besides its orbital, spin, and isospin angular momenta. To compute this contribution toward the angular momentum of the dyon, let us find the expressions on generalized electric and magnetic fields using the following ansatz of Julia and Zee:

$$V^{a}_{i}(r) - \varepsilon_{aij}(\hat{r})^{j}[K(r) - 1]\frac{1}{q^{*}r}, \tag{2.24}$$

$$V^{a}_{o}(r) - \varepsilon_{aij}(\hat{r})^{a}J(r)\frac{1}{q^{*}r}, \tag{2.25}$$

where i and o indicate space and time directions, a is an $SU(2)$ vector index (internal direction) and

$$\hat{r} = \frac{r}{r}.$$

In the static gauge, generalized electric and magnetic fields corresponding to the generalized field tensor given by (2.10) may be written as

$$E_k^a = G_{ok}^a = \partial_k V_o^a \mid q^* \varepsilon^{abc} V_{kb} V_{oc}, \tag{2.26}$$

and

$$B_k^a = \varepsilon_{ijk} G_{ij}^a = \partial_j V_i^a - \partial_i V_j^a + q^* \varepsilon^{abc} V_{jb} V_{ic} \quad (i \neq j \neq k), \tag{2.27}$$

showing that these generalized fields are non-Abelian in nature having external as well as internal components. In the Abelian limit these fields reduce to the usual electromagnetic fields in the simple static gauge. Using the values of $V_i^a(r)$ and $V_o^a(r)$ given by Eqs. (2.24) and (2.25) into (2.26) and (2.27), we get the following expressions for the generalized electric and magnetic fields associated with the non-Abelian dyon:

$$E_j^a = \frac{(\hat{r})}{q^*} \frac{\partial}{\partial r^j} \left(\frac{J(r)}{r} \right) + \frac{2}{q^*} \left\{ \frac{[K(r) - 1] J(r)}{r^2} \right\} (\hat{r})^a (\hat{r})_j \tag{2.28}$$

$$B_j^a = \frac{(\hat{r})_j}{q^*} \frac{\partial}{\partial r_a} \left[\frac{K(r) - 1}{r} \right] - \frac{r^b}{q^*} \frac{\partial}{\partial r^b} \left[\frac{K(r) - 1}{r} \right] (\hat{r})^a (\hat{r})_j$$

and

$$- \frac{(\hat{r})^a (\hat{r})_b}{q^*} \left[\frac{K(r) - 1}{r} \right]^2. \tag{2.29}$$

The external fields may be obtained by taking $r > r_0$, where r_0 is the radius of dyonic core. For large r the function $K(r) \to 0$ and

$$J(r) = b + Mr, \tag{2.30}$$

where b and M are positive constants having the dimensions of charge and mass, respectively. Then the external electric and magnetic fields reduce to the following forms in the asymptotic limit:

$$E_j^a = \frac{3b}{q^* r^2} (\hat{\mathbf{r}})^a (\hat{\mathbf{r}})_j - \frac{2M}{q^* r} (\hat{\mathbf{r}})^a (\hat{\mathbf{r}})_j, \tag{2.31}$$

$$B_j^a = \frac{-(\hat{\mathbf{r}})_j (\hat{\mathbf{r}})^a}{q^*} \frac{1}{r^2}. \tag{2.32}$$

For vanishing M, these fields correspond to pointlike, massless Abelian dyon with electric charge $3b/q^*$, and the magnetic charge $1/q^*$.

The internal fields may be obtained by taking $r < r_0$ and the following values of the functions $J(r)$ and $K(r)$ in the limit $r \to 0$:

$$J(r) \to 0 \qquad K(r) \to 1. \tag{2.33}$$

Then the internal fields at vanishingly small distance from the dyonic centre may be written as follows from Eqs. (2.28) and (2.29):

$$E_j^a = \frac{(\hat{r})^a}{rq^*}\frac{\partial}{\partial r^j}(J(r)) \qquad B_j^a = \frac{(\hat{r})_j}{rq^*}\frac{\partial K(r)}{\partial r_a}. \tag{2.34}$$

Since the derivatives of the functions $J(r)$ and $K(r)$ exist for small values of r, these fields are well defined near the dyonic centre. However, exactly at the dyonic centre, these fields blow up and hence there is no well-defined $U(1)$ charge symmetry at the dyonic centre. Similar conclusions have been derived earlier (Yoneya, 1984) by a different approach. At a finite distance $r < r_0$ in the dyonic core we may choose

$$J(r) - C_1 r^2 \qquad K(r) - C_2 r^2 + 1,$$

where C_1 and C_2 are constants. Substituting these values in Eqs. (2.28) and (2.29) we get the following expressions for the internal fields:

$$E_j^a = \frac{2C_1(\hat{r})^a(\hat{r})_j}{q^*} + \frac{2C_1C_2r^2}{q^*}(\hat{r})^a(\hat{r})_j, \tag{2.35}$$

and

$$B_j^a = \frac{2C_2}{q^*}(\hat{r})^a(\hat{r})_j - \frac{C_2^2(\hat{r})^a(\hat{r})_j r^2}{q^*}. \tag{2.36}$$

For vanishingly small r, these equations reduce to Eq. (2.34) with $J(r)$ and $K(r)$ given by Eq. (2.35). The angular-momentum operator of the non-Abelian dyon may be obtained as follows by using Eq. (2.18):

$$J_i = \varepsilon_{ijk}\int d^3r\, r^j T_0^k. \tag{2.37}$$

The residual part of the angular momentum (i.e. field contribution) may be obtained in the region outside (external) as well as inside (internal) of the dyonic core by identifying the field (2.29) as external field for $r > r_0$ with internal position vector and unit vector as $\vec{x}^a$ and $\hat{x}^a$ and as internal field for $r < r_0$ with external position vector and unit vector as $\mathbf{r}$ and $\hat{r}_j$. Then we get

$$\mathbf{J}_{\text{ext}} = \hat{r} \otimes (\mathbf{E}^a \otimes \mathbf{B}^a),$$

and

$$\vec{J}_{\text{int}} = \{\vec{x} \times (\vec{E}_j \times \vec{B}_j)\}, \tag{2.38}$$

where the symbol $\times$ denotes cross product in the internal space and $\otimes$ is the cross product in the external space. Substituting the expressions (2.28) and (2.29) in these equations we get

$$
\begin{aligned}
J_{\text{ext}} = {} & \frac{r \times (\hat{r}_{\mathrm{E}} \times \hat{r}_{\mathrm{B}})}{(q^*)^2} \left\{ \frac{-(K(r)-1)J(r)}{r^2} [3\hat{x}^a \cdot \hat{r}_{\mathrm{E}} - x^a] r \cdot \nabla \left(\frac{K(r)-1}{r} \right) \right. \\
& \times (\hat{r}_a \cdot \hat{r}_{\mathrm{B}}) - \frac{(K(r)-1)^3 J(r)}{r^4} [(3\hat{x}^a \cdot \hat{r}_{\mathrm{E}}) x_a - 1] + \frac{(K(r)-1)J(r)}{r^2} \\
& \left. \times\, [(3\hat{x}^a \cdot \hat{r}_{\mathrm{E}} - x_a)] \nabla_a \left(\frac{K(r)-1}{r} \right) \right\} + \frac{\mathbf{r}\left[\nabla\left(\frac{J(r)}{r}\right) \times \hat{r}_B\right]}{(q^*)^2} \\
& \times \left\{ \frac{-(K(r)-1)^2}{r^2} - \hat{x}_a^a \left(\mathbf{r} \cdot \nabla \left(\frac{K(r)-1}{1} \right) \right) (\hat{x}^a \cdot \hat{r}_{\mathrm{B}}) \right. \\
& \left. + \,\hat{x}^a \nabla_a \left(\frac{K-1}{r} \right) \right\}
\end{aligned}
\tag{2.39}
$$

$$
\begin{aligned}
\vec{J}_{\text{int}} = {} & \frac{\vec{x} \times (\vec{x}_E \times \vec{x}_B)}{(q^*)^2} \left\{ \frac{(-K(r)-1)J(r)}{r^2} [3\hat{x}_E \cdot \hat{r}_j - \hat{r}_j] \vec{x} \cdot \vec{\nabla} \left(\frac{K(r)-1}{r} \right) \right. \\
& \left. \times (\hat{x}_B \cdot \hat{r}^j) - \left(\frac{(K(r)-1)J(r)}{r^1} \right) [(3\hat{x}_E \cdot \hat{r}_j) r^j - 1] \right\} \\
& + \frac{\vec{x} \times \left(\vec{x}_E \times \nabla\left(\frac{K(r)-1}{r}\right)\right)}{(q^*)^2} \left\{ \frac{(K(r)-1)J(r)}{r^2} [3(\hat{x}_E \cdot \hat{r}_j) r^j - 1] \right. \\
& \left. + \, r_j \frac{\partial}{\partial r_j} (J(r)/r) \right\},
\end{aligned}
\tag{2.40}
$$

where $\hat{r}_E$ and $\hat{r}_B$ are unit vectors in the direction of electric and magnetic fields.

Substituting the limiting values of $J(r)$ and $K(r)$ given by Eq. (2.33) into Eq. (2.37), we get the following expression for the components of the internal angular momentum operator

$$
J_i^a = \frac{1}{r} \frac{\partial J(r)}{\partial r^j} \frac{\partial K(r)}{\partial x_a}, \tag{2.41}
$$

which blows up at $r \to 0$. In other words, if the core radius is assumed to be zero, then the density of the radial angular momentum is infinite. It corresponds to the infinite coulomb energy of the charge on the core. Thus, when a fermion scatters from the core of the dyon and changes its charge, the lost charge must be deposited on the dyon core (in order to maintain overall charge conservation) and the core must neutralize itself by some sort of pair creation process. This pair creation effect leads to baryon-number–nonconservation in the presence of non-Abelian dyon. The dyonic core has remarkable abilities to absorb baryon and lepton numbers with no loss in the energy.

3. FREMION-NUMBER BREAKING IN THE PRESENCE OF DYON

We consider an $SU(2)$ gauge theory with a Higgs triple φ^a and two left-handed fermionic doublets $\psi^{(s)}(s = 1, 2$ is the "flavor" index) throughout this paper. We always use a euclidean formulation of the field theory, so the action functional is

$$S = S_{A,\varphi} + S\psi \tag{3.1}$$

$$\begin{aligned} S_{A,\varphi} &= \int dt \left\{ \int d^3x \left[-\frac{1}{2e^2} S_P F_{\mu\nu}^2 + \frac{1}{4} S_P (D_\mu \varphi)^2 \right. \right. \\ &\quad \left. \left. + \lambda (S_P \varphi^2 - 2c^2)^2 \right] - M_{\text{dyon}} \right\} \\ &= \text{bosonic part (including mass of dyon)} \end{aligned} \tag{3.1a}$$

$$\begin{aligned} S_{A,\psi} &= -i \int d^3 x\, dt \sum_{s=1,2} \overline{\psi}^{(s)} \gamma_L^\mu (\partial_\mu + A_\mu) \psi^{(s)} \\ &= \text{fermionic part of the action.} \end{aligned} \tag{3.1b}$$

Lagrangian density can be written as

$$L = \left[-\frac{1}{2e^2} S_P F_{\mu\nu}^2 + \frac{1}{2} S_P (D_\mu \varphi)^2 + \lambda (S_P \varphi^2 - 2c^2)^2 \right] - M_{\text{dyon}}$$

where

$$M_{\text{dyon}} = \frac{1}{2} \int g^{\mu\nu} \vec{V}_\mu^* \cdot J_\mu \, d^3x$$

Since we are interested in the dyon sector, it is convenient to normalize the zero-point energy so that the dyon energy is equal to zero,

$$E_{\text{dyon}} = 0. \tag{3.2}$$

According to this prescription, the last term (the dyon mass) on the r.h.s. of (3.1a) is added to the standard bosonic part of the action of the Georgi–Glashow $SU(2)$ model. The "left-handed γ matrices" are defined by the following relations,

$$\gamma^\mu \frac{1 - \gamma^5}{2} = \begin{pmatrix} 0 & \gamma_L^\mu \\ 0 & 0 \end{pmatrix}$$

or, explicitly,

$$\gamma_L^0 = 1 \qquad \gamma_L^i = i\sigma_i,$$

σ_i being Pauli matrices. The matrix notation for V_μ and φ,

$$V_\mu = \frac{e}{2i} V_\mu^a \tau^a \qquad \varphi = \varphi^a \tau^a,$$

is used in (3.1a) and (3.1b).

The Higgs field φ develops a nonzero vacuum expectation value, so that in the unitary gauge

$$\langle\varphi\rangle_{\text{vac}} - c\tau_3$$

and only the third component V_μ^3 (photon) remains massless. In this gauge the fermionic sector consists of four massless left-handed fermions $\psi_-^{(s)} \equiv \psi_1^{(s)}$ and $\psi_+^{(s)} \equiv \psi_2^{(s)}$ carrying the electromagnetic charge $[-\frac{1}{2}e]$ and $[+\frac{1}{2}e]$, respectively (the lower index 1, 2 is the $SU(2)$ group one). The gauge-invariant current of sth fermion,

$$J_\mu^{(s)} = \overline{\psi}^{(s)}\gamma_L^\mu\psi^{(s)},$$

has the anomalous divergences

$$\partial_\mu J_\mu^{(s)} = \frac{1}{32\pi^2}\varepsilon_{\mu\nu\lambda\rho}S_P F_{\mu\nu}F_{\lambda\rho}. \tag{3.3}$$

The Julia–Zee dyon solution is

$$\begin{aligned} V_i^a &= \varepsilon_{aij}(\hat{r})^j\frac{[K(r)-1]}{|q|r}, \\ V_o^a &= \varepsilon_{aij}(\hat{r})^a\frac{J(r)}{|q|r}, \\ \varphi_a &= (\hat{r})_a H(r)/|q|^r, \end{aligned} \tag{3.4}$$

where i and o indicates space and time directions, a is an $SU(2)$ vector index, (i.e. internal direction) and

$$\hat{r} = \frac{\vec{r}}{r},$$

under the boundary conditions

$$H(0) = J(0) = 0 = K(0) - 1 = 0 \qquad H(\infty) = K(\infty) = 0. \tag{3.5}$$

$H(r)$ and $K(r)$ are exponentially small at $r \gg c^{-1}/q$, $r \gg c^{-1}/\lambda$. Throughout this paper we are primarily interested in the dynamical properties of fermions far from the dyon centre, i.e. we assume the limit

$$c \to \infty \qquad K \to 0 \tag{3.6}$$

to be taken whenever possible (otherwise the function $K(r)$ will be explicitly indicated). Note that in this limit dyon size vanishes

$$r_{\text{D}} \to 0. \tag{3.7}$$

Throughout this paper we treat the configuration (3.3) as a classical background one, though we do not assume the perturbations to be small. Thus the generating functional for the fermionic Green functions in the presence of a dyon

$$Z^{\text{dyon}}[\bar{\xi}, \xi] = \left\langle \exp\left[\int (\bar{\xi}\psi + \bar{\psi}\xi)\, d^4x\right]\right\rangle^{\text{dyon}}$$

(we omit the flavor index s whenever possible as well as the summation over s) is represented by the functional integral

$$Z^{\text{dyon}}[\bar{\xi}, \xi] = \int dV_\mu\, d\varphi \exp[-S_{V,\varphi} + \text{ gauge fixing term} + \text{ghost term}]$$
$$\times \int \prod_{s=1}^{2} d\psi^{(s)}\, d\bar{\psi}^{(s)} \exp\left[-S_{V,\psi} + \int (\bar{\xi}\psi + \bar{\psi}\xi) d^4x\right] \quad (3.8)$$

with the following boundary conditions:

$$V_\mu(x, t) \to V_\mu^a(x, t)$$
$$\varphi(x, t) \to \varphi^a(x) \quad t \to \pm\infty. \quad (3.9)$$

Equation (3.3) implies ('t Hooft, 1976a) that the change of the sth flavor is equal up to the winding number of the gauge field (1),

$$\Delta N^{(s)} = -q.$$

We first show that there exist configurations of the bosonic fields, obeying the boundary conditions (3.9) and having $q = -1$ and that, as opposed to the vacuum sector, in the dyon sector the action $S_{V,\varphi}$, for these configurations can be arbitrarily close to zero. This means that the supression factor $\exp(-\text{const}/e^2)$ does not appear in fermion-number–nonconerving matrix element. Consider the configuration

$$V_O = \tau^a n^a a_0(r, t)/i,$$
$$V_i = \tau^a n^a n_i a_1(r, t)/i + V_i^a, \quad (3.10)$$
$$\varphi = \varphi^a,$$

where $a_0(r, t)$ and $a_1(r, t)$ obey

$$a_0(r, \pm\infty) - a_1(r, \pm\infty) - 0. \quad (3.11)$$

The action functional for this configuration reads

$$S_{V,\varphi} = \frac{4\pi}{e^2} \int_0^\infty dr \int_{-\infty}^{\infty} dt\big[(\partial_t a_1 - \partial_r a_0)^2 r^2 + 2K^2\big(a_0^2 + a_1^2\big)\big] \quad (3.12)$$

and the winding number (1) reads

$$q = \frac{1}{\pi} \int_0^\infty dr \int_{-\infty}^{+\infty} dt\{\partial_r[a_0(1\ K)^2]\partial_t[a_1(1\ K)^2]\} \quad (3.13)$$

i.e., in virtue of (3.5) and (3.11),

$$q = \frac{1}{\pi} \lim \int_{-\infty}^{+\infty} dt\, a_0(r, t). \tag{3.14}$$

Note that the last expression can be obtained from (3.13) in the limit (3.6) and (3.7) only if $a_0(r, t)$ satisfies

$$a_0(r = 0, t) = 0. \tag{3.15}$$

An explicit example of a configuration obeying (3.11) and (3.15) and having $q = -1$ is

$$\begin{aligned} a_0(\rho/r, t) &= -\partial_r \rho(r, t), \\ a_0(\rho/r, t) &= -\partial_t \rho(r, t), \end{aligned} \tag{3.16}$$

with

$$\rho(r, t) - \frac{1}{2} \log\left[\mu_1^2(r^2 + t^2) + 1\right] - \frac{1}{2\varepsilon}\left[\mu_1^2(r^2 + t^2) + 1\right]^{-\varepsilon}, \tag{3.17}$$

where μ_1 is some mass scale and ε is a positive number. The value of action for the fields (3.10) and (3.16) is

$$S_{V,\varphi} = \frac{5\pi^2}{3e^2}\varepsilon(1 + O(\varepsilon) + O(\mu_1/c))$$

and can be arbitrarily small for small ε and μ_1, Q.E.D.

In the vacuum sector, fermion-number–breaking matrix elements are also suppressed by negative powers of c ('t Hooft, 1976b). This suppression occurs because the zero fermion modes far from the instanton are proportional to $\lambda_{\text{inst}}^{-3/2}$, and the instanton size λ_{inst} is bounded from above by c^{-1}. Thus, it is instructive to investigate the zero fermion modes in the external field (3.10) in order to find their c dependence. For the sake of convenience we consider the fields a_0 and a_1 of the form (3.16). Since the external field is spherically symmetric, it is natural to choose a spherically symmetric anstaz for the zero modes. A most general spherically symmetric fermionic field has the following form:

$$\psi_{\alpha l}^{(0)}(x, t) = \left(\sqrt{8\pi r}\right)^{-1} \exp\left(\int_{\infty}^{r} K(r')\frac{dr'}{r'}\right) \chi_{\alpha l}(x, t) \tag{3.18}$$

where $\alpha = 1, 2$ and $l = 1, 2$ are Lorentz and gauge group indices respectively, and

$$\chi_{\alpha l}(x, t) = \varepsilon_{\alpha l}\chi_1(r, t) - i\tau_{\alpha\beta}^{a}\varepsilon_{\beta l} n_a \chi_2(r, t). \tag{3.19}$$

Introducing the compact notation

$$\chi = \begin{pmatrix} \chi_1 \\ \chi_2 \end{pmatrix},$$

we obtain the following equation for the zero mode

$$\chi^{(0)} : \left[(\partial_t + i\tau_2 \partial_r \rho) + i\tau_2(\partial_r - i\tau_2 \partial_t \rho) + \frac{K}{r}(\tau_1 + i\tau_2) \right] \chi^{(0)} = 0 \qquad (3.20)$$

In order that the solution be nonsingular at $r = 0$, one should impose the boundary condition

$$(\tau_1 + i\tau_2)\chi^{(0)}(r - 0) - 0 \qquad (3.21)$$

The solution of Eqs. (3.20) and (3.21) is

$$\chi^{(0)}(r, t) = N \, \exp\{-\rho(r, t)\} \begin{pmatrix} 1 \\ 0 \end{pmatrix}, \qquad (3.22)$$

where N is a normalization factor. Since the zero modes (3.18) and (3.22) is square-integrable near $x = 0$ in the limits (3.6) and (3.7), the factor N is independent of c far from the dyon center. This implies that the fermion-number–breaking matrix elements in the presence of a dyon are not suppressed by negative powers of c. Note that, as is seen from (3.17). The zero mode has the following asymptotic behaviour as $r^2 + t^2 \to \infty$,

$$\psi \approx r^{-1}(r^2 + t^2)^{-1/2},$$

so that its norm $\int \psi^+ \psi \, d^4x$ is logarithmically divergent in the infrared region. This fact is in complete analogy to the Schwinger model (Krasnikov *et al.*, 1980a,b; Rothe and Swieca, 1979; Schroer, 1978).

The nature of the configuration (3.10) is most transparent in the unitary gauge of Arafune *et al.* (1975) and Englert and Windey (1976). Performing the transformation to this gauge, we obtain in the limits (3.6) and (3.7)

$$\begin{aligned} V_o^{(u)} &= \frac{1}{i}\tau_3 a_0, \\ V_i^{(u)} &= \frac{1}{i}\tau_3 a_1 + V_i^{\mathrm{D}} \frac{\tau_3 q}{2i}, \\ \phi^{(u)} &= c\tau_3, \end{aligned} \qquad (3.23)$$

where V_i^{D} is the Dirac expression for the vector-potential of the dyon carrying. From (3.23) it is clear that the configuration (3.10) is purely electromagnetic. Moreover, the electromagnetic field of the configuration is just the dyon one, while the electric field $E_i = (i/e)S_P K_{0i}\tau_3$ is

$$E_i = \frac{4n_i}{e}(\partial_0 a_1 - \partial_1 a_0)$$

and is directed along the electromagnetic field, so that $HE \neq 0$, $EH \neq 0$ (produced by dyon)(c.f. Introduction section); So this argument implies strong fermion-number–breaking in the presence of dyon.

4. MASSLESS LEFT-HANDED FERMION IN THE FIELD OF A DYON

This section is devoted to the study of left-handed massless fermion in the external field (3.10), in the limits (3.6) and (3.7). It is convenient to introduce the operator of total angular momentum (Dereli *et al.*, 1976):

$$M_i = -i\varepsilon_{ijk}x_j\partial_k + \frac{1}{2}\sigma_i + \frac{1}{2}\tau_i. \tag{4.1}$$

this operator commutes with Dirac operator $\mathbb{D}$,

$$\mathbb{D} = \gamma_L^{\mu}(\partial_\mu + V_\mu),$$

as well as with the operators τn and σn. The angular part of the Dirac operator,

$$\mathbb{D}_\Omega \equiv ir\sigma_k(\delta_{kl} - n_k n_l)(\partial_l + V_l) - i\sigma_k n_k,$$

commutes with τn and anticommutes with σn,

$$[\sigma_i n_i, \mathbb{D}_\Omega]_+ = 0. \tag{4.2}$$

It is a matter of straightforward calculation to verify the following identity:

$$\mathbb{D}_\Omega^2 = M^2. \tag{4.3}$$

There exist two eigenfunctions of M with zero eigenvalue, namely $\varepsilon_{\alpha l}$ and $\tau^a_{\alpha\beta}\varepsilon_{\beta l}n_\alpha$ (α and l are the Lorentz and gauge group indices respectively), and $4(2J+1)$ eigenfunctions $\psi_{JM\delta\nu}$ of M^2, which can be chosen to satisfy

$$\begin{aligned} M^2\psi_{JM\delta\nu} &= J(J+1)\psi_{JM\delta\nu} \quad J = 1, 2\ldots, \\ M^3\psi_{JM\delta\nu} &= M\psi_{JM\delta\nu} \quad M = 0, \pm 1, \ldots, \pm J, \\ \tau n\psi_{JM\delta\nu} &= \delta\psi_{JM\delta\nu} \quad \delta = \pm 1, \\ \sigma n\psi_{JM\delta\nu} &= \nu\psi_{JM\delta\nu} \quad \nu = \pm 1. \end{aligned} \tag{4.4}$$

the function $\psi_{JM\delta\nu}(J \neq 0)$ from a set of functions, which is complete in the subspace with $J \neq 0$ and orthonormal on a sphere. Thus, the fermion field ψ can be decomposed in the following way:

$$\psi(x,t) = \psi^{(0)}(x,t) + \frac{1}{r}\sum_{JM\delta}\sum_{\nu} u_\nu^{JM\delta}(r,t)\psi_{JM\delta\nu}(\Theta, \Phi), \tag{4.5}$$

where $\psi^{(0)}(x,t)$ is given by (3.18) and (3.19) (but the field χ need not satisfy the Dirac equation). It is convenient to introduce the compact notation

$$u^{JM\delta} = \frac{1 - i\tau}{\sqrt{2}}\begin{pmatrix} u_{+1}^{JM\delta} \\ u_{-1}^{JM\delta} \end{pmatrix}$$

and to rewrite the fermionic part of the action (3.1b) in the following form:

$$S_{V,\psi} = S_{J=0} + \sum_{J\neq 0}\sum_{M,\delta} S_{JM\delta}, \tag{4.6}$$

where

$$S_{J=0} = \int dr\, dt\, \overline{\chi} D_{J=0}\chi, \tag{4.7}$$

$$S_{SM\delta} = -i\int dr\, dt\, u^{-JM\delta} D_{J\delta} u^{JM\delta}, \tag{4.8}$$

$$D_{J-0} = \partial_t - i\tau_2 a_0 + i\tau_2(\partial_r - i\tau_2 a_1), \tag{4.9}$$

$$D_{J,\delta} = \partial_t - i\delta a_0 - i\tau_2(\partial_r - i\delta a_1) + \frac{\sqrt{J(J+1)}}{r}\tau_1 \tag{4.10}$$

(the limit (3.6) and (3.7) is assumed).

The form of the vector potential (3.10), the decomposition (4.5) and the actions $S_{J=0}$ and $S_{JM\delta}$ are invariant under the following transformation:

$$\begin{aligned} \chi &\to e^{i\tau_2\beta}\chi, \\ u^{JM\delta} &\to e^{i\delta\beta}u^{JM\delta}, \\ a_0 &\to a_0 + \partial_t\beta \qquad a_1 \to a_1 + \partial_t\beta, \end{aligned} \tag{4.11}$$

where $\beta(r, t)$ is some real function. The transformation (4.11) is a special case of gauge transformation, the gauge function

$$g(x, t) = \exp\lfloor i\tau^a n^a \beta(r, t)\rfloor \tag{4.12}$$

being spherically symmetric. For this gauge function to be nonsingular at $r = 0$, the function β should vanish at the origin,

$$\beta(r = 0, t) = 0. \tag{4.13}$$

According to the decomposition (4.5), the functional measure in (3.8) can be rewritten in the following form:

$$\prod_{s=l}^{2}\prod_{x,t} d\psi^{(s)} d\overline{\psi}^{(s)} = \prod_{r,t}\prod_{s=l}^{2}\left[d\chi^{(s)} d\overline{\psi}^{(s)} \prod_{JM\delta} du^{(s)JM\delta} d\overline{u}^{(s)JM\delta}\right] \tag{4.14}$$

Thus, the functional integral over fermions in the external field (3.10) reduces to an infinite product of functional integrals over the two-dimensional fermionic fields $\chi(r, t)$ and $u^{JM\delta}(r, t)$ (defined on a half-plane), the relevent action functionals being given by (4.7) and (4.8).

We begin the discussion of the above action functional by deriving the Green function of zero-angular-momentum fermions. Since the Dirac operator (4.9) in the limit of a pointlike dyon is ill defined at $r = 0$ [ ], we consider the full operator for the field χ (cf.(3.20)),

$$D^{\text{full}}_{J=0} - \partial_t - i\tau_2 a_0 + i\tau_2(\partial_r - i\tau_2 a_1) + \frac{F}{r}(\tau_1 - i\tau_2). \tag{4.15}$$

the Green function G $(rt; r't')$ obeys the following equation:

$$D^{\text{full}}_{J=0} G(rt; r't') = \delta(r - r')\delta(t - t'). \tag{4.16}$$

To derive the boundary condition for G, we assumme for simplicity the function F to be step function,

$$F(r) = \theta(r - r_{\text{D}})$$

where r_{D} is the dyon radius. At the end of our derivation we shall take the limit (3.7). We also assume that the functions a_0 and a_1 are finite and smooth at $r = 0$. The standard arguments of the theory of differential equations leads to the following behaviour of G $(rt; r't')$ near the origin $r = 0$:

$$\left.\begin{aligned} G_1(rt; r't') &= 0(1), \\ G_2(rt; r't') &= 0(r), \end{aligned}\right\} \tag{4.17}$$

where $G_{1,2} = \frac{1 \neq \tau_3}{2} G$.

From Eqs. (4.15) and (4.16) it follows that G $(rt; r't')$ is continuous at $r = r_{\text{D}}$, and from (4.17) in the limit (3.7) we obtain the following boundary condition:

$$(1 - \tau_3) G(0t; r't') = 0. \tag{4.18}$$

Note that in terms of the field $\chi(r, t)$ this boundary condition corresponds to (3.21). Thus, in the limits (3.6) and (3.7) the Green function of the field χ obeys the equation

$$D_{J=0} G(rt; r't') = 0. \tag{4.19}$$

$$V_o = 0 \tag{4.20}$$

or

$$a_0 = 0 \tag{4.21}$$

and consider the function $a_1(r, t)$ obeying the boundary condition (3.11). In this case G$(rt; r't')$ can be obtained in the closed form, namely,

$$G(rt; r't') = \exp[-\sigma(r, t) + \sigma(r', t') + i\tau_2\gamma(r, t)] G_0(rt; r't') \exp[i\tau_2\gamma(r', t')], \tag{4.22}$$

where

$$\sigma(r, t) = \int_0^\infty dr'' \int_{-\infty}^\infty dr'' [Đ(r - r'', t - t'') + Đ(r + r'', t - t'')] \times \partial_{t''} a_1(r'', t''), \tag{4.23}$$

$$\gamma(r,t) - \int_{-\infty}^{t} \partial_r \sigma(r,t'')\,dt''. \tag{4.24}$$

Here $\text{Đ}(r,t)$ is the propagator of the two-dimensional massless scalar field (the inverse two-dimensional Laplacian),

$$\text{Đ}(r,t) = \frac{1}{4\pi}\log \mu_2^2(r^2+t^2) \tag{4.25}$$

(μ_2 is an arbitrary mass scale (Klaiber, 1968; Wightman, 1966), and G_0 is the solution of the "free" equation

$$(\partial_t + i\tau_2\partial_r)G_0(rt;r't') = \delta(r-r')\delta(t-t'),$$

obeying the boundary condition

$$(1-\tau_3)G_0(0t;r't') = 0.$$

Explicitly

$$\begin{aligned} G_0(rt;r't') &= (\partial_t - i\tau_2\partial_r)[\text{Đ}(r-r',t-t') + \text{Đ}(r+r',t-t')\tau_3] \\ &= \frac{1}{2\pi}\left[\frac{(t-t') - i\tau_2(r-r')}{(r-r')^2+(t-t')^2} + \frac{(t-t') - i\tau_2(r+r')}{(r+r')^2+(t-t')^2}\right]. \end{aligned} \tag{4.26}$$

Note that the definitions (4.23) and (4.24) imply

$$\partial_r\sigma(0,t) = 0 \qquad \gamma(0,t) = 0. \tag{4.27}$$

Now we turn to the discussion of the action (4.8). In this case we cannot find the exact Green function of the operator $D_{J,\delta}$, so we develop perturbation theory around $a_0 = a_1 = 0$. The free propagator G^J corresponding to the action (4.8) obeys the equation

$$\left(\partial_t - i\tau_2\partial_r + \frac{b(J)}{r}\right)G^J(rt;r't') = \delta(r-r')\delta(t-t'), \tag{4.28}$$

where

$$b(J) = \sqrt{J(J+1)}.$$

It is straightforward to prove that the solution of (4.28) has the following form

$$G^J = \begin{pmatrix} \partial_t \Re_{b^2+b} & \left(\partial_r - \frac{b}{r}\right)\Re_{b^2-b} \\ \left(-\partial_r - \frac{b}{r}\right)\Re_{b^2+b} & \partial_t \Re_{b^2-b} \end{pmatrix}, \tag{4.29}$$

where the function $\Re_k(rt; r't')$ obeys the equation

$$\left(\partial_t^2 + \partial_r^2 + \frac{k}{r^2}\right)\Re_k(rt; r't') = \delta(r - r')\delta(t - t'). \tag{4.30}$$

Using the properties of the Legendre function $Q_m(z)$ listed in Appendixes A and B, one can verify that the solution of (4.30) is

$$\Re_k(rt; r't') = -\frac{1}{2\pi} Q_{d(k)}\left[1 + \frac{(r - r')^2 + (t - t')^2}{2rr'}\right], \tag{4.31}$$

where

$$d(k) = \sqrt{k + \frac{1}{4}} - \frac{1}{2}. \tag{4.32}$$

The free propagator (4.29) vanishes as $(r - r')^2 + (t - t')^2$ tends to infinity as well as at $r = 0$ (see Appendixes A and B).

To conclude this section we summarize the analogus properties of the left-handed fermion in an external gauge field of the form

$$\begin{aligned} \tilde{V}_o &= \frac{1}{i}\tau_3 a_0(r, t), \\ \tilde{V}_i &= \frac{1}{i}\tau_3 n_i a_1(r, t), \\ \tilde{\varphi} &= c\tau_3. \end{aligned} \tag{4.33}$$

This field is purely electromagnetic and differs from the unitary gauge configuration (3.23) by the Dirac vector potential V_i^D. In this case the angular momentum operator is the standard one,

$$\tilde{M}_i - \varepsilon_{ijk} \int d^3r\, r^j T_0^k + \frac{1}{2}\sigma_i, \tag{4.34}$$

and the decomposition analogous to (4.5) reads

$$\psi(x, t) = \frac{1}{r} \sum_{r,k,\delta} \sum_{\nu} \nu_\nu^{n,k,\delta}(r, t) \tilde{\psi}_{nk\delta\nu}(\Theta, \Phi), \tag{4.35}$$

where $\tilde{\psi}_{nk\delta\nu}$ and the eigen function of $\tilde{M}^2$, $\tilde{M}_3$, τ_3 and σn with the eigenvalues

$$n - \frac{1}{2}(n = 1, 2, \ldots), \quad k\left(k = \pm\frac{1}{2}, \ldots, \pm\left(n - \frac{1}{2}\right)\right), \quad \delta(\delta = \pm 1),$$
$$\text{and } \nu(\nu = \pm 1),$$

respectively. The fermionic action in the enternal field (4.33) can be rewritten as ⌈cf.(4.6)⌉

$$S_{V,\psi} = \sum_{nk\delta} \tilde{S}_{nk\delta},$$

where

$$\tilde{S}_{nk\delta} = -i \int dr\ dt \overline{\nu}^{nk\delta}(r,t) \tilde{D}_{n,\delta} \nu^{nk\delta}(r,t), \tag{4.36}$$

and

$$\tilde{D}_{n,\delta} = \partial_t - i\delta a_0 - i\tau_2(\partial_r - i\delta a_1) + \frac{n}{r}\tau_1.$$

The free operator $\tilde{G}^n$ corresponding to the action (4.36) can be found in the same way as G^J.

$$\tilde{G}^n = \begin{pmatrix} \partial_t \mathfrak{R}_{n^2+n} & \left(\partial_r - \frac{n}{r}\right) \mathfrak{R}_{n^2-n} \\ \left(-\partial_r - \frac{n}{r}\right) \mathfrak{R}_{n^2+n} & \partial_t \mathfrak{R}_{n^2-n} \end{pmatrix}. \tag{4.37}$$

5. THE ZEROTH-ORDER APPROXIMATION

We describe an approximation for evaluating matrix elements of zero-angular-momentum fermionic fields in the presence of a dyon, i.e. the matrix element of the following form:

$$W(r_1 t_1, \ldots, r'_N t'_N) = \langle \chi(r_N t_N), \ldots, \chi(r_N t_N) \overline{\chi}(r'_1 t'_1), \ldots, \overline{\chi}(r'_N t'_N) \rangle^{\text{dyon}} \tag{5.1}$$

Using the representations (which are inverse to (4.5) and (3.18))

$$\chi_1^{(s)} = (8\pi)^{-1/2}\, r \int \varepsilon_{\alpha 1} \Psi_{\alpha 1}^{(s)}(x,t) \sin\Theta\, d\Theta\, d\Phi$$

$$\chi_2^{(s)} = i(8\pi)^{-1/2}\, r \int \varepsilon_{l\beta} \tau^a_{\beta\alpha} n^a \Psi_{\alpha 1}^{(s)}(x,t) \sin\Theta\, d\Theta\, d\Phi \tag{5.2}$$

one can relate the matrix elements (5.1) to the matrix elements of the initial fields $\Psi^{(s)}$ in the presence of a dyon.

The functional integral representation (5.1) for the matrix elements (3.8) can be rewritten in the following way:

$$W(r_1 t_1, \ldots, r'_N t'_N) = \int dV_\mu\, d\varphi \exp[-S_{V,\varphi} - \hat{S}[V, \varphi; r_1 t_1, \ldots, r'_N t'_N] + \text{gaugefixing terms} + \text{ghost terms}]$$

where the fields $V_{\mu,\varphi}$ obey the boundary conditions (3.9) and

$$e^{-\hat{S}} = \int \prod_{s-1}^{2} d\overline{\Psi}^{(s)} a\psi^{(s)} e^{-SV,\psi} \chi(r_1 t_1), \ldots, \chi(r'_N t'_N).$$

We search for the minimum of the effective action $S_{V,\varphi} + \hat{S}$ and assume that, to the lowest order in e^2 and c^{-1}, the matrix element (5.1) is

$$W\big(r_1 t_1, \ldots, r'_N t'_N\big) = \exp[-(S_{V,\varphi} + \hat{S})_{\min}].$$

We also assume that the fields V and φ realizing this minimum take the form (3.10), where the field $a_1(r, t)$ obeys the boundary condition (3.11) (we are still proceeding in the temporal gauges (4.20) and (4.21)). Under the above assumptions the fermionic contribution to the effective action $\hat{S}$ takes a particularly simple form

$$\begin{aligned} \hat{S} = {} & -2 \sum_{J\neq 0} \sum_{\delta} (2J+1) \log\ \mathrm{Det}[iD_{J\delta}(a_1)] - 2 \log\ \mathrm{Det}[iD_{J=0}(a_1)] \\ & + \sum_{p=1}^{N} [\sigma(r_p, t_p) - \sigma(r'_p, t'_p)] - \log \Bigg\{ \exp\Bigg[\sum_{p=1}^{N} iT_2^{(p)} \gamma(r_p, t_p) \\ & + \sum_{p'=1}^{N} iT_2^{(p')} \gamma(r'_p, t'_p) \Bigg] W^{(O)}(r_1 t_1, \ldots, r'_N t'_N) \Bigg\} \end{aligned} \tag{5.3}$$

where the operators $D_{J=0}$ and $D_{J,\delta}$ are defined by (4.9) and (4.10), σ and γ are defined by (4.23) and (4.24), and $W^{(0)}(r_1 t_1, \ldots, r'_N t'_N)$ is the “free” (no interaction with a_1) matrix element (5.1), i.e. the Wick expansion of (3.1) with the pairing (4.26). Equation (5.3) is a direct consequence of (4.5)–(4.7) and (4.22); the factor 2 in the first two terms on the r.h.s. of (5.3) comes from the summation over the flavours, while the factor $(2J + 1)$ in the first term of the r.h.s. of (5.3) comes from the summation over the third component of angular momentum.

Now we make another assumption that will be justified in our forthcoming paper. We assume that the first term on the r.h.s. of (5.3) is negligible. Since the a_1 dependences of the third the fourth terms in (5.3) are explicit, we only have to evaluate the second term. This can be done in the same way as in the Schwinger model (Schwinger, 1962), so we only sketch the derivation. It is convenient to adopt the following unified notation. By $\xi_\iota(\iota = 0, 1)$ we denote the coordinates in the (t, r) half-plane:

$$\xi_0 = t \qquad \xi_1 = r, \tag{5.4a}$$

so that

$$\xi^2 \equiv \xi_\iota \xi_\iota = r^2 + t^2 \qquad d^2\xi = dr\, dt. \tag{5.4b}$$

The variation of the second term on the r.h.s. of (5.3) with respect to the variation of a_1 is

$$\delta(-2\log\ \text{Det}\ i\ D_{J=0}) = -2\int d^2\xi\ Sp\ G(\xi,\xi)\delta a_1(\xi). \tag{5.5}$$

From the explicit expressions (4.22) and (4.26), it follows that the contribution of the second (nonsingular) term on the r.h.s. of (4.26) vanishes. Using the point-splitting regularization,

$$G(\xi,\xi) = \frac{1}{2}\lim[G(\xi/\varepsilon) + G(\xi/-\varepsilon)],$$

$$G(\xi/\varepsilon) = \exp\left(i\int_{\xi}^{\xi+\varepsilon} a_\iota(\xi')d\xi'_\iota\right) G(\xi,\xi+\varepsilon),$$

which is invariant under the gauge transformation (4.11), we obtain

$$Sp\ G(\xi,\xi) = -\frac{1}{\pi}\partial_t\sigma(r,t),$$

where σ is defined by (4.23). From (5.5) we get

$$\delta(-2\log\ \text{Det}\, i\, D_{J=0}) = \frac{2}{\pi}\int dr\ dt\ \partial_t\sigma\cdot\delta a_1 = -\frac{2}{\pi}\int \sigma\delta\left[\left(\partial_r^2+\partial_t^2\right)\sigma\right]dr\ dt. \tag{5.6}$$

The last expression has been obtained by integration by parts with the use of (4.27) (this is another way to understand the necessity of the boundary condition (4.27)). Finaly, from (5.6) we find

$$2\log\ \text{Det}\, i\, D_{J=0} = -\frac{1}{\pi}\int \sigma\left(\partial_r^2\ \middle|\ \partial_t^2\right)\sigma\ dr\ dt \tag{5.7}$$

In terms of the variable σ, the action $S_{V,\varphi}$ can be rewritten as (see (3.12); we still take the limits (3.6) and (3.7))

$$S_{V,\varphi} = \frac{4\pi}{e^2}\int dr\ dt\left[\left(\partial_r^2+\partial_t^2\right)\sigma\right]^2\cdot r^2 \tag{5.8}$$

so the effective action $S_{V,\varphi} + \hat{S}$, within our approximation, is at most quadratic in σ [the last term in (5.3) is, in fact, linear in γ and hence in σ] and the quadratic part is the sum of (5.7) and (5.8),

$$S_2(\sigma) = S_{V,\varphi} - 2\log\ \text{Det}\, i\, D_{J=0} = \frac{1}{2}\int \sigma(r,t)L_{r,t}\sigma(r,t)\,dr\ dt, \tag{5.9}$$

where

$$L_{r,t} = -\frac{2}{\pi}\left(\partial_r^2+\partial_t^2\right) + \frac{8\pi}{e^2}\left(\partial_r^2+\partial_t^2\right)r^2\left(\partial_r^2+\partial_t^2\right) \tag{5.10}$$

We conclude that, within our approximation, the matrix elements (5.1) are equal to

$$W\big(r_1 t_1, \ldots, r'_N t'_N) = \exp\left\{-\left(S_2 + \int \sigma j \, dr \, dt\right)_{\min}\right\} \times W^0(r_1 t_1, \ldots, r'_N t'_N\big), \tag{5.11}$$

where

$$\int dr \, dt \, \sigma j = \sum_{p-1}^{N} \left[\sigma(r_p, t_p) - \sigma(r'_p, t'_p)\right] + \sum_{p-1}^{N} i \tau_2^{(p)} \gamma(r_p, t_p) + \sum_{p'=1}^{N} i \tau_2^{(p')} \gamma(r'_p, t'_p) \tag{5.12}$$

is the linear term in (5.3). To find the explicit expression for the exponential in (5.11), it is sufficient to determine the Green function $\wp(rt; r't')$ of the operator (5.10). This function obeys the following equation

$$L_{rt} \wp(rt; r't') = \delta(r - r')\delta(t - t'). \tag{5.13}$$

Since the function

$$\sigma(r, r \mid j) - - \int \wp(rt; r't') j(r', t') dr' \, dt',$$

realizing the minimum of $S_2 + \int j\sigma$, should obey the boundary condition (4.27), the defining equation (5.13) should be supplemented by the following boundary condition:

$$\partial_r \wp(0t; r't') = 0 \tag{5.14}$$

As is clear from (4.30) and (A.15), the solution of (5.13) and (5.14) is

$$\wp(rt; r't') = \frac{1}{2}\pi \left[\Re_{e^2/4\pi^2}(rt; r't') - Đ(r - r', t - t') - Đ(r + r', t - t')\right] \tag{5.15}$$

where the function $Đ$ is defined by (4.25). Equations (5.11), (5.12), and (5.15), are sufficient to evaluate the matrix elements (5.1) within our approximation. Rather than present explicit expressions that are somewhat complicated, we prefer to describe the functional integral fit for these matrix elements. From (5.11), (5.12), and (5.15) we find

$$W(r_1 t_1, \ldots, r'_N t'_N) = \int \prod_{r,t,s} d\chi_0^{(s)} d\overline{\chi}_0^{(s)} d\Sigma \, d\eta \times \exp(-S_\Sigma - S_\eta - S\chi_0)\chi(r_1, t_1) \tag{5.16}$$

where the fields Σ, η, and χ_0 are defined on a half-plane $\{r \in (0, \infty), t \in (-\infty, +\infty)\}$ and obey the boundary conditions

$$\partial_r \Sigma(0, t) - \partial_r \eta(0, t) - (1 - \tau_3)\chi_0(0, t) - 0.$$

The effective actions are

$$S_\Sigma = -\frac{1}{2}\int dr\ dt \Sigma \left(\partial_r^2 + \partial_t^2 - \frac{e^2}{4\pi r^2}\right)\Sigma,$$

$$S_\eta = +\frac{1}{2}\int dr\ dt\ \eta\left(\partial_r^2 + \partial_t^2\right)\eta,$$

$$S_{\chi_0} = \int dr\ dt_{\bar{\chi}_0} D_{J=0}(a = 0)\chi_0,$$

with D_{j-0} defined by (4.9), and

$$\chi^{(s)}(r, t) = \exp\left[-\tilde{\sigma}(r, t) + i\tau_2 \int_{-\infty}^{t} \partial_r \tilde{\sigma}(r, t')\, dt'\right]\chi_0^{(s)},$$

with

$$\tilde{\sigma}(r, t) - \sqrt{\frac{1}{2}\pi}[\Sigma(r, t) + \eta(r, t)].$$

Note that the integrals (5.16) are Gaussian and the propagators of the fields Σ and η are

$$\mathfrak{R}_{e^2/4\pi^2}(rt; r't') \qquad [-Đ(r - r', t - t') - Đ(r + r', t - t')],$$

respectively, while the propagator of χ_0 is given by (4.26). Note also, that the fit (5.16) is analogous to the (euclidean) functional integral counterpart of the VLS-like operator solution (Lowenstein and Swieca, 1971; Velo, 1967) of the γ^5 analogue (Krasnikov *et al.*, 1979) of the Schwinger model, transformed to the temporal gauge.

6. DISCUSSION

We want to conclude this whole paper with these four points.

6.1. No Well-Defined Charge Symmetry at the Dyonic Core

Since it has been demonstrated (Mandelstam, 1976, 1979; Rubakov, 1981a; ’t Hooft, 1978) that the helicity-conserving and charge-mixing boundary conditions to be imposed on fermionic fields at monopole core violate the charge superselection rule. This problem may possibly find a solution by establishing the formation of chiral condensate, allowing the flipping of helicity and proving an understanding of the role of dyon as catalyst in baryon-number–nonconserving processes. This problem of chiral condensation, at the dyonic core requires the explicit field solutions in the interior region of dyon. We have investigated the extended structure of non-Abelian dyon in Section 2 by constructing suitable Lagrangian density (2.17) and angular-momentum tensor (2.18) in non-Abelian gauge theory of dyons. Since an Abelian dyon moving in the generalized field of another dyon carries a residual angular momentum (field contribution) besides its orbital and spin angular momenta. Keeping in mind this fact and Julia-zee dyon solutions (2.24) and (2.25) we have constructed the residual part of the angular momentum (i.e. field contribution) in the interior as well as exterior regions of dyon (2.39) and (2.40). We can conclude by Eq. (2.41) that when a fermion scatters from the core of the dyon and changes its charge, the lost charge must be deposited on the dyonic core (in order to maintain overall charge conservation) and the core must neutralize itself by some sort of pair creation process. This pair creation effect leads to baryon number non conservation in the presence of non-Abelian dyon has been undertaken in rest of the part of the paper. The dyonic core has remarkable abilities to absorb baryon and lepton numbers at no loss in the energy.

6.2. Relation to the θ Vacuum Structure

Since the vacuum structure of the gauge theories is most apparent in the temporal gauge (4.20) (Callen *et al.*, 1976; Jackiw and Rebbi, 1976), it is convenient to proceed in this gauge. From (5.15) we find that the temporal gauge saddle-point field $a_1^{t,-}$ can be represented as

$$a_1^{t,-}(r,t;r_1,t_1) = \pi\theta(t-t_1)\delta(r-r_1) - \partial_r^2 \int_{-\infty}^{t} \mathfrak{R}_{e^2/4\pi^2}(rt';r_1t_1)\,dt' - \partial_t \mathfrak{R}_{e^2/4\pi^2}(rt;r_1t_1) \tag{6.1}$$

From (A.16) it follows that the last term vanishes as $t \to \mp\infty$ so the field $a_1^{t,-}$ interpolates between the following two configurations:

$$a_1^{t,-}(r, t = -\infty/r_1) = 0, \tag{6.2a}$$

$$a_1^{t,-}(r, t = \infty/r_1) = 0 = \partial_r \Omega(r/r_1), \tag{6.2b}$$

where

$$\Omega(r/r_1) = \pi\theta(r-r_1) - \partial_r \int_{-\infty}^{+\infty} \mathfrak{R}_{e^2/4\pi^2}(r,t';0,r_1)\,dt'$$

The field (6.2b) is a pure gauge (see (4.11)); from (A.15) and (A.16) we find the following asymptotics of the gauge function Ω:

$$\Omega(0/r_1) = 0 \qquad \Omega(\infty/r_1) = \pi. \tag{6.3}$$

Now we recall the fact that the gauge transformation (4.11) in terms of the initial fields A_μ, φ, and ψ is just the usual gauge transformation with the gauge function (4.12). Thus, the saddle-point configuration (i.e., the configuration (3.10) with $a^{t,-}$ substituted for a) interpolates between the fields

$$V_i(t=\infty) = V_i^a \qquad \varphi(-\infty) = \varphi^a,$$
$$V_i(t=+\infty) = g_\Omega V_i^a g_\Omega^{-1} + g_\Omega \partial_i g_\Omega^{-1},$$
$$\varphi(t=+\infty) = g_\Omega \varphi^a g_\Omega^{-1} = \varphi^a,$$

where

$$g_\Omega = \exp(i\tau^a n^a \Omega). \tag{6.4}$$

From (6.3) we conclude that the gauge function (6.4) has just the same form as that considered in Callen *et al.* (1976) Jackiw and Rebbi (1976) and its topological number is equal to -1. The arguments as those of Callen *et al.* (1976) Crewether (1981) Jackiw and Rebbi (1976) show that the vector $U[g_\Omega]\,|M, 0\rangle$ ($U\lceil g_\Omega \rceil$ being the operator of the gauge transformation with the gauge function (6.4)), which is the gauge transform of the perturbation theory dyon state $|M, 0\rangle$, carries one unit of each flavor. This could also be anticipated, since the operator $U[g_\Omega]$ carries one unit of each flavor, as follows from the considerations of Callen *et al.* (1976) Crewether (1981) Jackiw and Rebbi (1976). The gauge-invariant dyon state is a linear superposition of the form

$$|M, \theta\rangle = \sum_{n=-\infty}^{+\infty} e^{in\theta} (U[g_\Omega])^n |M, 0\rangle; \tag{6.5}$$

this is another way to understand the fermion-number breaking in the presence of a dyon. In fact, the heuristic arguments of Section 3 are simplified in the temporal gauge; indeed, the unboundedness from below (by any positive number) of the action (3.12) can be established by the Derric-like (Rubakov, 1981) time rescaling.

6.3. The Unitary Gauge

The particle content of the theory with the action (3.1) is most apparent in the unitary gauge. In this gauge it makes sense to consider the matrix element $\langle \varepsilon_{\alpha\beta} \Psi_{+\alpha}^{(1)} \Psi_{-\beta}^{(2)} \rangle^{\text{dyon}}$ ($\alpha, \beta = 1, 2$ are Lorentz indices, the fields $\Psi_\pm^{(s)}$ are defined in Section 3). Since the operator $\Psi_+^{(1)} \Psi_-^{(2)}$ carries one unit of each flavor, the nonzero contributions to this matrix element come from the unitary gauge configurations with the winding number equal to -1, in particular, from the field (3.23) and (3.16) (or (3.23)). The latter contribution is proportional to the zero fermion modes in the external fields (3.23) and (3.16), namely, it is proportional to

$$\varepsilon_{\alpha\beta} \Psi_{+\alpha}^u \Psi_{-\beta}^u, \tag{6.6}$$

where the zero mode Ψ^u is just the zero modes (3.18) and (3.22) transformed to the unitary gauge. Performing this gauge transformation (the corresponding gauge function is described, e.g., in Arafune *et al.* (1975) Englert and Windey (1976) far from the dyon center, we obtain

$$\Psi^u_+ = B(r,t)\begin{pmatrix} \sin\frac{1}{2}\Theta e^{-i\Phi} \\ -\cos\frac{1}{2}\Theta \end{pmatrix},$$
$$\Psi^u_- = B(r,t)\begin{pmatrix} \cos\frac{1}{2}\Theta \\ \sin\frac{1}{2}\Theta e^{i\Phi} \end{pmatrix}, \tag{6.7}$$

where Θ and Φ are polar angles and

$$B(r,t) = \frac{N}{\sqrt{8\pi}}\frac{e^{-\rho(r,t)}}{r}.$$

Note that Ψ^u_- is the CP conjugate of Ψ^u_+. From (6.6) and (6.7) we conclude that $\langle \varepsilon_{\alpha\beta}\Psi^{(1)}_{+\alpha}\Psi^{(2)}_{-\beta}\rangle^{\text{dyon}} \neq 0$, i.e. the Adler-Bell–Jackiw anomaly gives rise to flavor-non-conserving and fermion-number–nonconserving transitions with charge conservation.

6.4. Baryon-Number Breaking in the Presence of Dyon

The dyon (Julia and Zee, 1975) of the $SU(5)$ grand unified theory coincides asymptotically with the ’t Hooft–Polyakov one for the $SU(2)$ group imbedded into $SU(5)$ in the following way:

$$T = \frac{1}{2}\text{diag}(0, 0, \tau, 0). \tag{6.8}$$

This dyon is fundamental in the sense that it is characterized by minimal electric and magnetic charge. With respect to $SU(2)$ specified by (6.8), the first-generation fermions form the following left-handed doublets (in the unitary gauge),

$$\begin{pmatrix} -\overline{u}^2 \\ u^1 \end{pmatrix}_L, \quad \begin{pmatrix} \overline{u}^1 \\ u^2 \end{pmatrix}_L, \quad \begin{pmatrix} d^3 \\ e^+ \end{pmatrix}_L, \quad \begin{pmatrix} e^- \\ -\overline{d}^3 \end{pmatrix}_L, \tag{6.9}$$

other being singlets. In (6.9) the superscripts 1, 2, and 3 are colour indices.

If u and d quarks and electrons were massless, the above arguments would be directly applicable to this case, so the matrix element

$$\langle u^1 u^2 d^3 e^- \rangle^{\text{dyon}} \tag{6.10}$$

would be nonzero, and coupling-constant- and unification-scale-independent. This conclusion remains unchanged if other (massive) generations are taken into account

(Rubakov, 1981b). The matrix element (6.10) corresponds to the process

$$p + \text{dyon} \to e^{+} + \text{dyon} + \text{everything}, \qquad (6.11)$$

and the arguments of the above paper imply that the cross-section of this paper is independent of the coupling constant and the unification scale, i.e. it is roughly $O(1GeV^{-2})$. Unfortunately, the above discussion is not quite decisive. First, electrons and quarks are massive. Naively, this seems to be inessential at distances in comparison to the compton wavelengths of electron and light quarks. However in the massive case the higher order corrections could destroy the boundary conditions (3.21) and (4.18), thus invalidating the above analysis. For example, the boundary conditions for fermions with extra magnetic moment (Kazama *et al.*, 1977; Kazama and Yang, 1977) differ from those given by (3.21). Second, in the above considerations we completely ignored gluon self-interaction. So, further investigations are required to establish the existence of processes like (6.11) and to estimate the cross-section of these processes.

In our forthcoming paper, the study of fermion-number–violating matrix element $\langle f(r_1, t_1)\rangle^{\text{dyon}} = \langle f\rangle^{\text{dyon}}$ of the operator $f(r,t) = \chi_1^{(1)}(r,t)\chi_1^{(2)}(r,t) + \chi_2^{(1)}(r,t)\chi_2^{(2)}(r,t)$ in presence of a dyon and density of the condensate of zero-angular-momentum fermions (an estimate of corrections), guided by the analogy with the Schwinger model (Krasnikov *et al.*, 1979; Nielsen and Schroer, 1977a,b) will be undertaken (In this section we further exploit the analogy used in Sections to discuss the fermion-number breaking in the presence of dyon.), which concludes that the approximation used is reasonable at least for the evaluation of Green functions of fermions with zero total angular momentum, including fermion-number–breaking Green functions.

APPENDIX A: LEGENDRE FUNCTION

In this appendix we summarize some relevent properties of the special functions.

The Legendre function $Q_m(x)$ obeys the following equation. (Abramowitz and Stegun, 1964; Bateman and Erdelyi, 1953; Gradshtein and Ryzhik, 1961):

$$(1-x^2)\frac{d^2Q_m}{dx^2} - 2x\frac{dQ_m}{dx^2} + m(m+1)Q_m = 0 \qquad (A1)$$

Its explicit expression for $m = 0$ is

$$Q_0(x) = -\frac{1}{2}\log\frac{x-1}{x+1}. \qquad (A2)$$

It has the following asymptotic behaviour as $x \to 1$ (Bateman and Erdelyi, 1953)

$$Q_m(x) = -\frac{1}{2}\log\frac{x-1}{2} - \Psi(m+1) + \Psi(1) + O[(x-1)\log(x-1)]. \qquad (A3)$$

From the representation (Abramowitz and Stegun, 1964; Bateman and Erdelyi, 1953; Gradshtein and Ryzhik, 1961)

$$Q_m(x) = 2^{-m-1}\pi^{1/2}\frac{\Gamma(m+1)}{\Gamma\left(m+\frac{3}{2}\right)}x^{-m-1}F\left(1+\frac{m}{2}, \frac{1+m}{2}; m+\frac{3}{2}; \frac{1}{x^2}\right)$$

where $F(\alpha, \beta; \gamma; x)$ is the hypergeometric function, it follows that

$$Q_m(x) = 2^{-m-1}\pi^{1/2}\frac{\Gamma(m+1)}{\Gamma\left(m+\frac{3}{2}\right)}x^{-m-1}(1+O(x^{-2})) \tag{A4}$$

at large X. Q^m can be also expressed as (Abramowitz and Stegun, 1964; Bateman and Erdelyi, 1953)

$$Q_m(x) = \left(\frac{1}{2}\pi\right)^{1/2}(x^2-1)^{-1/4}\frac{\Gamma(m+1)}{\Gamma\left(m+\frac{3}{2}\right)}\left[x-(x^2-1)^{1/2}\right]^{m+1/2}$$
$$\times F\left(\frac{1}{2}, \frac{1}{2}; m+\frac{1}{2}; -\frac{x-(x^2-1)^{1/2}}{2(x^2-1)^{1/2}}\right).$$

Using the Stirling formula,

$$\Gamma(m) = e^{-m+m\log m}\, m^{-1/2}(2\pi)^{1/2}(1+O(m^{-1})),$$

as well as the definition of the hypergeometric series, we find at large m and x fixed

$$Q_m(x) = \left(\frac{1}{2}\pi\right)^{1/2} m^{-1/2}(x^2-1)^{-1/4}\left[x-(x^2-1)^{1/2}\right]^{m+1/2} \tag{A5}$$

Now we derive the asymptotic expansion of $Q_m(x)$ as $m \to \infty$, which is uniformly valid at $1 < x < \infty$. We use the method described by Thorne (1957) and consider the function $y(\tau)$ defined by

$$y(\tau) = \left(\frac{\sinh\tau}{\tau}\right)^{1/2} Q_m(\cosh\tau).$$

From (A1) we obtain the following equation for $y(\tau)$:

$$\frac{d^2y}{d\tau^2} + \frac{1}{\tau}\frac{dy}{d\tau} - \Lambda^2 y + w(\tau)y = 0, \tag{A6}$$

where

$$\Lambda - m + \frac{1}{2} \quad w(\tau) - \frac{1}{4}(\sinh^{-2}\tau - \tau^{-2}). \tag{A7}$$

We search for the solution of (A6) in the form of asymptotic series

$$y(\tau) = K_0(\Lambda\tau)\sum_{n=0}^{\infty}\frac{T_n(\tau)}{\Lambda^{2n}} - \frac{K_1(\Lambda\tau)}{\Lambda}\sum_{n=0}^{\infty}\frac{R_n(\tau)}{\Lambda^{2n}}, \tag{A8}$$

where K_m are modified Bessel functions. Inserting (A8) into (A6), we obtain the following recurrent relations:

$$R_n(\tau) = -\frac{1}{2}\int_0^{\tau}\left[T_n''(\tau') + \frac{T_n'(\tau')}{\tau'} + w(\tau')T_n(\tau')\right]d\tau',$$

$$T_{n+1}(\tau) = -\frac{1}{2}\int_0^{\tau}\left[R_n''(\tau') - \frac{R_n'(\tau')}{\tau'} + \frac{R_n(\tau')}{(\tau')^2} + w(\tau')R_n(\tau')\right]d\tau' \quad \text{(A9)}$$

By comparing the behaviours of $y(\tau)$ and $K_0(\Lambda\tau)$ at small τ, namely (see (A3) and (Abramowitz and Stegun, 1964; Gradshtein and Ryzhik, 1961)

$$y\tau = -\log\tau + O(1),$$

$$K_0(\Lambda\tau) = -\log\tau + O(1),$$

we find that

$$T_0 = 1. \quad \text{(A10)}$$

Equations (A9) and (A10) are sufficient to determine the unknown functions T_u and R_u. Note that at small τ

$$R_n = O(\tau) \quad n \geq 0,$$

$$T_n = O(\tau^2) \quad n > 0,$$

Thus, the desired expansion is

$$Q_m(\cosh\tau) = \left(\frac{\tau}{\sinh\tau}\right)^{1/2}\left\{K_0\left[\left(m+\frac{1}{2}\right)\tau\right]\sum_{n=0}^{\infty}\frac{T_n(\tau)}{\left(m+\frac{1}{2}\right)^{2n}} - \frac{K_1\left\lfloor\left(m+\frac{1}{2}\right)\tau\right\rfloor}{m+\frac{1}{2}}\sum_{n=0}^{\infty}\frac{R_n(\tau)}{\left(m-\frac{1}{2}\right)^{2n}}\right\} \quad \text{(A11)}$$

Note that the asymptotic expansion (A11) is a particular case of Thorne's (1957) and is uniformly valid in the region $0 < \tau < \infty$. Performing the change of variables, $\tau = z/(m + 1/2)$, we find another asymptotic expansion,

$$Q_m\left(\cosh\frac{z}{m+\frac{1}{2}}\right) = K_0(z)\left\{1 + \frac{z^2}{\left(m+\frac{1}{2}\right)^2}\sum_{n=0}^{\infty}\frac{\tilde{T}_n(z)}{\left(m+\frac{1}{2}\right)^{2n}}\right\} - \frac{K_1(z)z}{\left(m+\frac{1}{2}\right)^2}\sum_{n=0}^{\infty}\frac{\tilde{R}_n(z)}{\left(m+\frac{1}{2}\right)^{2n}}. \quad \text{(A12)}$$

where

$$\tilde{R}_n = T_n = O(1) \quad z \to 0 \quad \text{(A13)}$$

APPENDIX B: THE FUNCTION $\mathfrak{R}_{\mathbf{k}}(rt;r't')$

$\mathfrak{R}_{\mathrm{k}}(rt;r't')$ is defined by (4.31) and (4.32). From (A1) and (A3) it follows that this function obeys (4.30). From (A3) we find

$$\mathfrak{R}_{\kappa}(rt;r't') = \frac{1}{4\pi}\log\frac{(r-r')^2+(t-t')^2}{4r^2} + \frac{1}{2\pi}\{\Psi[d(\kappa+1)]-\Psi(1)\} + O\{[(r-r')^2+(t-t')^2]\log[(r-r')^2+(t-t')^2]\} \quad \text{(A14)}$$

at small $(r-r')^2+(t-t')^2$. Equation (A4) yields

$$\mathfrak{R}_{\kappa} = \alpha(\kappa)\left[\frac{rr'}{r'^2+(t-t')^2}\right]^{1/d(\kappa)} \qquad r\to 0 \quad \text{(A15)}$$

as well as

$$\mathfrak{R}_{\kappa} = \alpha(\kappa)\left[\frac{(r-r')^2+(t-t')^2}{rr'}\right]^{-1-d(\kappa)} \qquad r^2+t^2\to\infty,$$

$$\alpha(\kappa) = -\frac{1}{2\sqrt{\pi}}\frac{\Gamma(1+d(\kappa))}{\Gamma\left(\frac{3}{2}+d(\kappa)\right)}. \quad \text{(A16)}$$

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